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## Public Debt Overhang in the Heterogeneous Agent Model \*

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### Abstract

In this paper, I demonstrate that expansionary fiscal policy associated with an increase in public debt can cause a persistent recession. I assume that entrepreneurs have borrowing constraints and that the government issues debt and collects tax from productive entrepreneurs. The government can also transfer resources to workers. Under this setting, an increase in public debt per se can enhance economic growth as it can compensate for the shortage of liquidity. However, output decreases as the transfer increases due to the income effect of the transfer on workers increasing the wage rate. Noticeably, both output and interest rates decline as the transfer and debt become larger. This result challenges the widely accepted view that the negative effect of expansionary fiscal policy on output should be the crowding-out effect that works through a hike in interest rates. It also implies that redistribution from workers to productive agents may enhance economic growth.

*Keywords:* Public debt overhang, Borrowing constraint, Redistribution

*JEL classification:* E32, E62, H20, H63.

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# 1 Introduction

The management of large and increasing public debt is one of the most urgent and primary policy issues for most developed nations. According to the Organisation for Economic Co-operation and Development (OECD), Japan's public debt is now 223 percent of the gross domestic product (GDP) and is increasing at an explosive speed (Johansson, et al. 2013). In the United States, the debt-GDP ratio is now 111 percent and is expected to reach 118 percent in 2050, mainly because of increasing interest costs, growing spending for social security, and the government's major health care programs (CBO 2013). The debt-GDP ratio of the Eurozone was 100 percent in 2013 and will stay at a very high level in the coming decades. In this environment, the causal link between debt and economic growth has become one of the important topics in academic and policy debates.

Policy debates on public debt and economic growth in Japan may be a valuable lesson for other developed nations that suffer from debt crises and aftershocks of the financial crisis. Japan has been suffering from low economic growth and accelerating accumulation of public debt for more than two decades. The fiscal policy was expansionary because of public works in the 1990s and increased social security spending in the 2000s. During the period, both long- and short-term interest rates have been low. Thus, it is a widely accepted view that the persistent stagnation was not caused by fiscal policy, because interest rates would have been high if the public sector had crowded out economic activities in the private sector. In the policy debate, economic growth and sustainability of public finance are considered as separate and even conflicting targets. However, if public finance caused the recession, these targets may not be conflicting and fiscal consolidation would enhance economic growth.

In this paper, I demonstrate that increases in public debt and transfer have causal links with a possibly persistent recession in a closed economy. I assume that entrepreneurs have borrowing constraints, whereas the government issues debt and collects tax from productive entrepreneurs and can also transfer resources to workers. Under this setting, output increases with increases in public debt and tax if there is no transfer and it de-

creases as transfers to workers increase. Noticeably, for a particular range of parameters, both output and interest rates decline as public debt and transfers become larger in this economy. This result challenges the widely accepted view that any negative effect of expansionary public finance on the economy should be the crowding-out effect that works through a hike in interest rates.

**Related Literature** There is growing empirical literature on the “public debt overhang,” that is, the depressive effect of public debt on economic growth. Reinhart, Reinhart, and Rogoff (2012) report 26 debt episodes in developed nations and argue that public debt may depress economic growth by 1 percent if and when it exceeds 90 percent of GDP. Checherita-Westphal and Rother (2012) and Baum, Checherita-Westphal, and Rother (2013) found a similar effect in the EU. In particular, Reinhart, Reinhart, and Rogoff (2012) point out that large public debt coexisted with not only low growth but also low interest rates in a significant number of episodes. Related to the literature on debt, Barro and Sala-i-Martin (2003) and Fischer (1991) show empirically that large government expenditures may depress economic growth. There are theoretical models in which public debt has a negative effect on economic growth (Saint-Paul 1992; Brauninger 2005; Arai, Kunieda, and Nishida 2012). They are endogenous growth models, and high interest rates are involved in the negative effect of debt that is substantially amplified by the assumed externality. Our model is complementary to these models in that it generates a negative effect of public finance on output without assuming any intrinsic externality.<sup>1</sup> In some theoretical literature, public debt is also shown to improve efficiency; for example, Diamond (1965), Woodford (1990), Holmstrom and Tirole (1998), Caballero and Krishnamurthy (2006), and Kocherlakota (2009). In these studies, public debt works as a liquid savings instrument for economic agents who suffer from shortage of liquidity, and thereby, improves economic efficiency. This liquidity-providing effect of

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<sup>1</sup>The non-Keynesian effect that fiscal consolidation enhances consumption and output is discussed empirically by Giavazzi and Pagano (1990) and theoretically by Perotti (1999), among many others. The non-Keynesian effect is by nature a temporary effect, whereas the negative effect in our model is persistent.

debt is present in our model too. In our theory, public debt per se may have a positive effect on the economy, but redistribution of resources from productive entrepreneurs to other agents by means of subsidy may depress output. Similar effects of redistribution policy are reported by Benabou (2002) and Seshadri and Yuki (2004). Grobety (2012) empirically reports the growth-enhancing effect of public debt.

The organization of this paper is as follows. In the next section, we present the model. In Section 3, we analyze the nature of steady-state equilibria under various fiscal-policy settings. In Section 4, we present our conclusion.

## 2 Model

The model represents a closed economy that can be viewed as a simplified version of Buera and Nicolini (2013) and a modified version of Kiyotaki (1998). There are a continuum of workers with measure 1, a continuum of high-productivity entrepreneurs with measure  $n$  ( $0 < n < 1$ ), and a continuum of low-productivity entrepreneurs with measure  $1 - n$ , and the government. The workers live forever. The entrepreneurs die with probability  $1 - \gamma$  in each period and  $1 - \gamma$  new entrepreneurs are born in each period. The productivity of an entrepreneur does not change until his death. The discount factor for workers is  $\beta$  ( $< 1$ ). The discount factor for entrepreneurs, conditional on survival, is  $\beta'$  ( $> \beta$ ). We assume that the unconditional discount factor for entrepreneurs is equal to that for workers, that is,  $\beta = \gamma\beta'$ . The entrepreneurs can produce the consumption good with Cobb–Douglas production technology; that is,  $y_t = A_t k_t^\alpha l_t^{1-\alpha}$ , where  $A_t$  is the productivity parameter. We assume that entrepreneurs are subject to the borrowing constraint that will be specified shortly.

**Fiscal policy:** We focus on the following fiscal policy:  $\{B_{t+1}, T_t, S_t\}$ , where  $B_{t+1}$  is the one-period bond redeemable at  $t + 1$ ,  $T_t$  is the lump-sum tax, and  $S_t$  is the lump-sum subsidy to the workers. In period  $t$ , the government issues bonds  $B_{t+1}/r_t$ , where  $r_t$  is

the gross market rate of interest, subject to the following budget constraint:

$$\frac{B_{t+1}}{r_t} + T_t = B_t + S_t.$$

The same amount of tax,  $T_t$ , is imposed on all (high- and low-productivity) entrepreneurs. No tax is imposed on workers. The budget constraint implies that the value of the current bond  $B_t$  satisfies

$$B_t = \sum_{j=0}^{\infty} \frac{T_{t+j} - S_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}},$$

where we define  $\prod_{s=0}^{-1} r_{t+s} = 1$ .

## 2.1 Workers

There is a unit mass of workers who can save but cannot borrow. They choose consumption,  $c'_t$ , labor supply,  $l_t$ , and bond holdings,  $b'_{t+1}/r_t$ , to maximize utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln c'_t + \omega \ln(1 - l_t)],$$

subject to the budget constraint,

$$c'_t + \frac{b'_{t+1}}{r_t} = w_t l_t + b'_t + S_t,$$

and the non-negativity constraint,  $b'_{t+1} \geq 0$ . The labor supply is determined by

$$w_t = \frac{\omega c'_t}{1 - l_t}.$$

If the non-negativity constraint is binding, the workers become hand-to-mouth workers who consume all their income during the same period and do not save. The condition for the workers to be hand-to-mouth is

$$\frac{c'_{t+1}}{c'_t} > \beta r_t$$

which I assume is always satisfied. The parameter value is chosen such that the labor supply equals  $\frac{1}{3}$  when the workers are hand-to-mouth, that is,  $\omega = 2$ .

## 2.2 Entrepreneur

There is a unit mass of entrepreneurs. Measure  $n$  of the entrepreneurs have productivity  $z$  and measure  $1 - n$  have productivity 1, where  $z > 1$ . We consider entrepreneurs with productivity  $z$  as being high productivity and those with productivity 1 as being low productivity. The terms “entrepreneur” and “firm” have been used interchangeably throughout the paper. At the end of every period,  $1 - \gamma$  entrepreneurs are randomly chosen to die and  $1 - \gamma$  new entrepreneurs are born at the beginning of the next period. Among the newborn entrepreneurs,  $(1 - \gamma)n$  have productivity  $z$  and  $(1 - \gamma)(1 - n)$  have productivity 1. The newborn entrepreneurs inherit the wealth of the dead entrepreneurs according to an exogenously given law that will be described later (see Assumption 2). The entrepreneur’s utility is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t, \quad (1)$$

where  $c_t$  is the entrepreneur’s consumption. An entrepreneur with productivity  $A \in \{1, z\}$  can produce output  $y_t$  from labor  $l_t$  and capital  $k_t$  by using the following production technology:

$$y_t = A k_t^\alpha l_t^{1-\alpha}.$$

We assume for simplicity that capital stock  $k_t$  fully depreciates after production of output. The budget constraint for an entrepreneur is as follows:

$$c_t + k_{t+1} - \frac{b_{t+1}}{r_t} \leq A k_t^\alpha l_t^{1-\alpha} - w_t l_t - b_t - T_t, \quad (2)$$

where  $b_{t+1}$  is the bond issued in period  $t$  and redeemed in period  $t + 1$ . Note that in cases where an entrepreneur purchases bonds issued by other entrepreneurs,  $b_{t+1}$  can be a negative number. We consider the following assumption pertaining to lack of commitment.

**Assumption 1** *An entrepreneur cannot commit to repayment of debt ( $b_{t+1}$ ). The creditors (or the bondholders) can seize  $\theta y_{t+1}$ , where  $y_{t+1}$  is the output and  $0 < \theta < 1$ , if the entrepreneur repudiates his debt.*

Under this assumption, a part of the output ( $\theta y_{t+1}$ ) works as collateral for the debt and the upper limit of the amount that can be borrowed. Thus, the entrepreneur faces the following borrowing constraint:

$$b_{t+1} \leq \theta A k_{t+1}^\alpha l_{t+1}^{1-\alpha}, \quad (3)$$

where  $l_{t+1}$  is the labor input in  $t + 1$  that is decided in period  $t + 1$ . As we will see later,  $l_{t+1}$  is a linear function of  $k_{t+1}$ . The optimization problem for an entrepreneur with productivity  $A$  is to choose  $\{c_t, k_{t+1}, b_{t+1}\}$  in period  $t$  and  $l_{t+1}$  in period  $t + 1$  to maximize his utility (1), subject to the budget constraint (2) and borrowing constraint (3).

**Low-productivity firms' problem:** Following Kiyotaki (1998), we limit our attention to the equilibrium where the borrowing constraint is sufficiently tight, such that the high-productivity firms cannot use up all the capital stock in the economy. In this case, the low-productivity entrepreneurs buy both bonds and capital stock, implying that the borrowing constraint is not binding for them.

$$\begin{aligned} (LP) \quad & \max \sum_{t=0}^{\infty} \beta^t \ln c_t, \\ \text{s.t.} \quad & a'_{t+1} = r_t(a'_t - c_t - T_t), \end{aligned}$$

where  $a'_t$  denotes the asset holdings of low firms. Because the borrowing constraint is not binding for the low firms, the marginal productivity of capital (MPK) for the low firms equals the market rate, that is,  $r_t = \alpha(l_t/k_t)^{1-\alpha}$ . Similarly, the marginal productivity of labor (MPL) for the low firms equals the wage rate, that is,  $w_t = (1 - \alpha)(k_t/l_t)^\alpha$ . These equations imply

$$r_t = \alpha \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}. \quad (4)$$



**High-productivity firms' problem:** Given  $r_t$ , the reduced form of the high firms' problem is written as follows.

$$(HP) \quad \max \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

$$\text{s.t. } a_{t+1} = R_t(a_t - c_t - T_t),$$

where  $a_t = zk_t^\alpha l_t^{1-\alpha} - w_t l_t - b_t$  and  $R_t$  is the gross rate of return for the high firms that is determined by the solution to the problem below. The problem is the maximization of the return on investment for the high firms, given the amount of remaining assets after consumption and tax payment ( $a_t - c_t - T_t$ ), as follows:

$$\max_{k_{t+1}, b_{t+1}} a_{t+1} = \pi(k_{t+1}, w_{t+1}) - b_{t+1},$$

$$\text{s. t. } \begin{cases} k_{t+1} - \frac{b_{t+1}}{r_t} \leq a_t - c_t - T_t, \\ b_{t+1} \leq \theta A k_{t+1}^\alpha l_{t+1}^{1-\alpha}, \end{cases}$$

where

$$l_{t+1} = \arg \max_l zk_{t+1}^\alpha l^{1-\alpha} - w_{t+1} l = \left( \frac{(1-\alpha)z}{w_{t+1}} \right)^{\frac{1}{\alpha}} k_{t+1},$$

$$\pi(k_{t+1}, w_{t+1}) = \max_l zk_{t+1}^\alpha l^{1-\alpha} - w_{t+1} l = \alpha z^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} k_{t+1}.$$

Given that the market rate of interest is given by (4), the solutions are

$$k_{t+1} = \hat{k}(a_t - c_t - T_t),$$

$$a_{t+1} = R_t(a_t - c_t - T_t),$$

where

$$\hat{k} = \frac{1}{1 - \frac{\theta}{\alpha} z^{\frac{1}{\alpha}}}, \quad (5)$$

$$R_t = \left( 1 - \frac{\theta}{\alpha} \right) z^{\frac{1}{\alpha}} r_t \hat{k}. \quad (6)$$

**Solution to entrepreneurs' problems:** The first order conditions (FOCs) for (HP) and (LP) imply that the consumption of a high entrepreneur who has wealth  $a_t$  is given

by

$$c_t = (1 - \beta) \left[ a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right],$$

and the consumption of a low entrepreneur who has wealth  $a'_t$  is given by

$$c'_t = (1 - \beta) \left[ a'_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right].$$

In the next period, the wealth of a high entrepreneur is given by

$$\frac{a_{t+1}}{R_t} = \beta \left[ a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}}, \quad (7)$$

and that of a low entrepreneur is given by

$$\frac{a'_{t+1}}{r_t} = \beta \left[ a'_t - \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}}. \quad (8)$$

### 2.3 Aggregate dynamics

Given the exogenous policy  $\{T_t, S_t\}_{t=0}^{\infty}$ , the dynamics are described as the evolution of two state variables, namely,  $(s_t, W_t)$ , where  $W_t$  is the total wealth in period  $t$  ( $W_t = \alpha Y_t + B_t$ ) and  $s_t$  is the high firms' share in the total wealth. The wealth of dead entrepreneurs is inherited by the newborn entrepreneurs in the same period by the following law:

**Assumption 2** *A newborn high-productivity entrepreneur inherits the wealth of a low-productivity entrepreneur who died in the same period.*<sup>2</sup>

This is almost equivalent to assuming that in every period,  $(1 - \gamma)n$  high firms change to low firms and the same measure of low firms become high firms, and that firm managers must exit when the productivities of their firms change. We define  $\hat{K}_{t+1}$  and  $\hat{L}_{t+1}$  as capital and labor used by the high firms, respectively, and  $K'_{t+1}$  and  $L'_{t+1}$  as those used by the low firms. Given  $(s_t, W_t)$ , the variables  $(r_t, R_t, W_{t+1}, s_{t+1}, K_{t+1}, \hat{K}_{t+1}, K'_{t+1}, \hat{L}_{t+1}, L'_{t+1}, L_{t+1},$  and  $w_{t+1})$  are calculated by the following system of 11 equations.<sup>3</sup>

<sup>2</sup>Here, we implicitly assume that  $0 < n < 0.5$ .

<sup>3</sup>This system of equations can be solved by the backward shooting method on the premise that the economy converges to the deterministic steady state.

$$r_t = \alpha \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}, \quad (9)$$

$$R_t = \frac{\left(1 - \frac{\theta}{\alpha}\right) z^{\frac{1}{\alpha}}}{1 - \frac{\theta}{\alpha} z^{\frac{1}{\alpha}}} r_t, \quad (10)$$

$$\begin{aligned} W_{t+1} = R_t & \left\{ \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\} \\ & + r_t \left\{ \beta \left[ (1 - s_t) W_t - (1 - n) \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right] + (1 - n) \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right\}, \end{aligned} \quad (11)$$

$$\begin{aligned} s_{t+1} W_{t+1} = \gamma R_t & \left\{ \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\} \\ & + (1 - \gamma) \frac{n}{1 - n} r_t \left\{ \beta \left[ (1 - s_t) W_t - (1 - n) \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right] + (1 - n) \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right\}, \end{aligned} \quad (12)$$

$$\begin{aligned} K_{t+1} = \sum_{j=1}^{\infty} \frac{S_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} & + \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \\ & + \beta \left[ (1 - s_t) W_t - (1 - n) \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}} \right] - n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} r_{t+s}}, \end{aligned} \quad (13)$$

$$\hat{K}_{t+1} = \hat{k} \left\{ \beta \left[ s_t W_t - n \sum_{j=0}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right] + n \sum_{j=1}^{\infty} \frac{T_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}} \right\}, \quad (14)$$

$$K'_{t+1} = K_{t+1} - \hat{K}_{t+1}, \quad (15)$$

$$\hat{L}_{t+1} = z^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1}{\alpha}} \hat{K}_{t+1}, \quad (16)$$

$$L'_{t+1} = \left( \frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1}{\alpha}} K'_{t+1}, \quad (17)$$

$$w_{t+1} = \frac{\omega[w_{t+1} L_{t+1} + S_{t+1}]}{1 - L_{t+1}}, \quad (18)$$

$$L_{t+1} = \hat{L}_{t+1} + L'_{t+1}. \quad (19)$$

## 2.4 Steady state

Because we are interested in analyzing the qualitative nature of the model, we focus on the steady-state equilibrium of this economy. Given the level of tax  $T_t = T$  and subsidy  $S$ , the steady state variables  $(B, r, R, W, s, K, \hat{K}, K', \hat{L}, L', L, w)$  can be determined as the solution to the following system of 12 equations.

$$B = \frac{r}{r-1}(T - S), \quad (20)$$

$$r = \alpha \left( \frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}}, \quad (21)$$

$$R = \frac{(1 - \frac{\theta}{\alpha}) z^{\frac{1}{\alpha}}}{1 - \frac{\theta}{\alpha} z^{\frac{1}{\alpha}}} r, \quad (22)$$

$$W = R \left\{ \beta \left[ sW - n \frac{RT}{R-1} \right] + n \frac{T}{R-1} \right\} + r \left\{ \beta \left[ (1-s)W - (1-n) \frac{rT}{r-1} \right] + (1-n) \frac{T}{r-1} \right\}, \quad (23)$$

$$sW = \gamma R \left\{ \beta \left[ sW - n \frac{RT}{R-1} \right] + n \frac{T}{R-1} \right\} + (1-\gamma) \frac{n}{1-n} r \left\{ \beta \left[ (1-s)W - (1-n) \frac{rT}{r-1} \right] + (1-n) \frac{T}{r-1} \right\}, \quad (24)$$

$$K = \frac{S}{r-1} + \beta \left[ sW - n \frac{RT}{R-1} \right] + n \frac{T}{R-1} + \beta \left[ (1-s)W - (1-n) \frac{rT}{r-1} \right] - n \frac{T}{r-1}, \quad (25)$$

$$\hat{K} = \hat{k} \left\{ \beta \left[ sW - n \frac{RT}{R-1} \right] + n \frac{T}{R-1} \right\}, \quad (26)$$

$$K' = K - \hat{K}, \quad (27)$$

$$\hat{L} = z^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} \hat{K}, \quad (28)$$

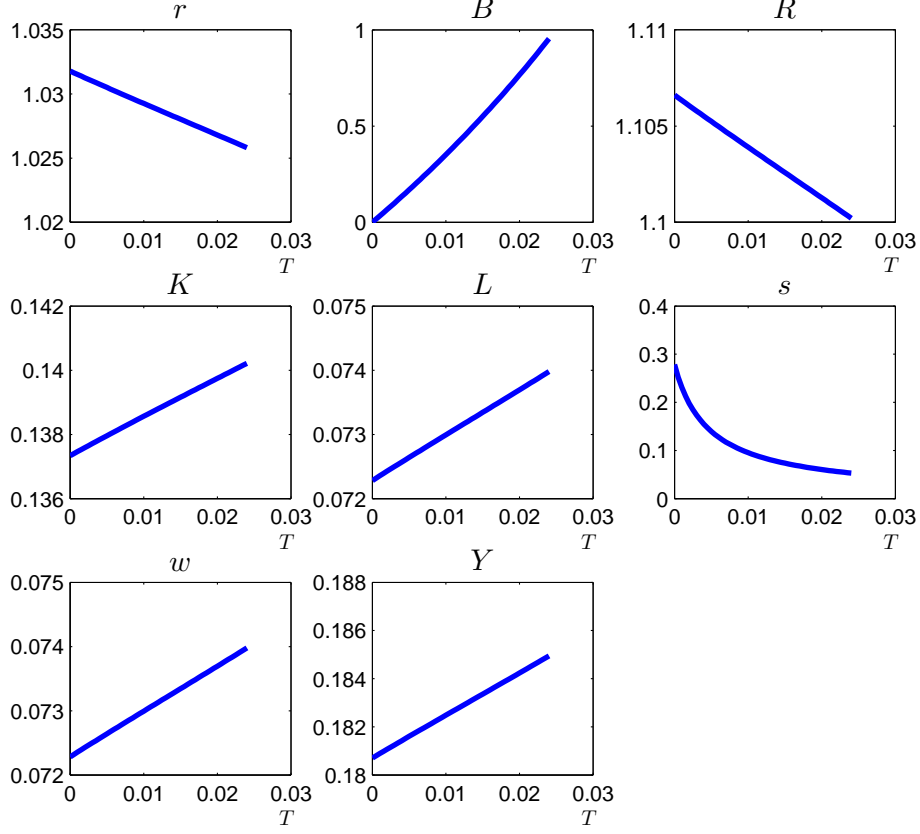
$$L' = \left( \frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} K', \quad (29)$$

$$w = \frac{\omega[wL + S]}{1-L}, \quad (30)$$

$$L = \hat{L} + L'. \quad (31)$$

The method to arrive at the solution is described in the Appendix.

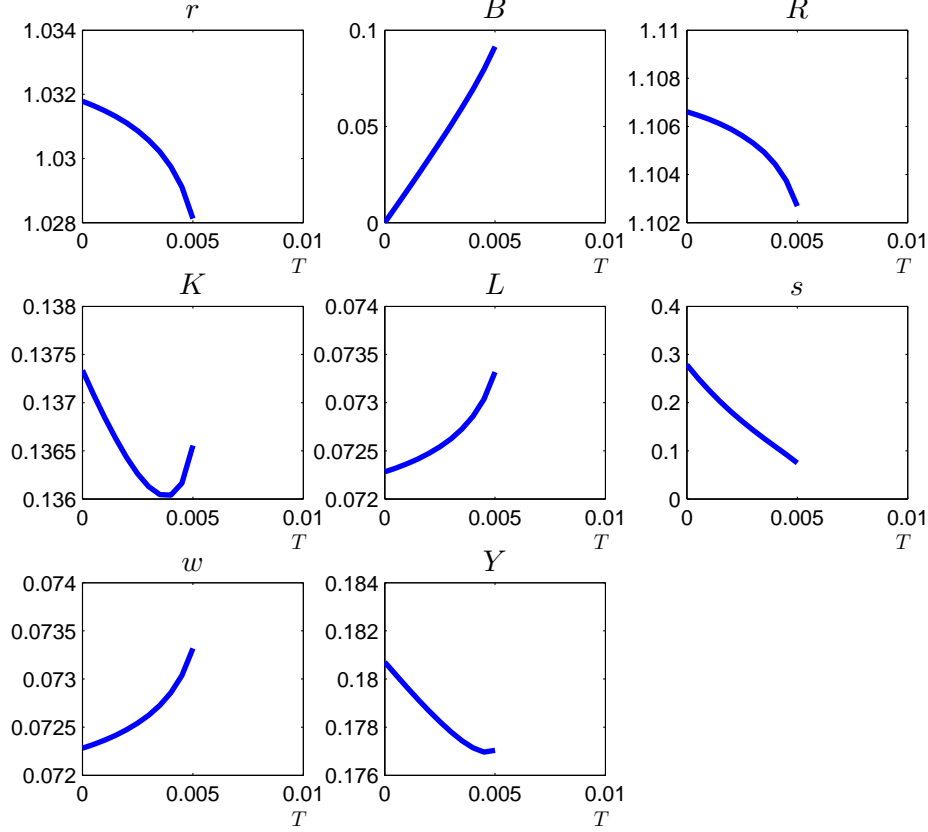
Figure 1: The steady-state equilibrium:  $S = 0$



### 3 Numerical simulations and discussions

Figure 1 shows the steady-state equilibrium in the case where  $S = 0$ . The parameter values are  $\alpha = 0.8$ ,  $\beta = 0.95$ ,  $\gamma = 0.95$ ,  $\theta = 0.1$ ,  $\omega = 1$ ,  $z = 1.05$ , and  $n = 0.01$ . This figure shows that as public debt and tax increase, output increases and the interest rate decreases. This result is qualitatively robust against perturbation of the parameter values. This result is consistent with the models in which public debt provides valuable liquidity (see, for example, Woodford 1990 and Holmstrom and Tirole 1998). The entrepreneurs are taxed and the tax revenue is transferred back to the bondholders, that is, the low entrepreneurs. In this economy, there is a shortage of privately provided bonds, because firms are subject to the lack of commitment. The government can mitigate this

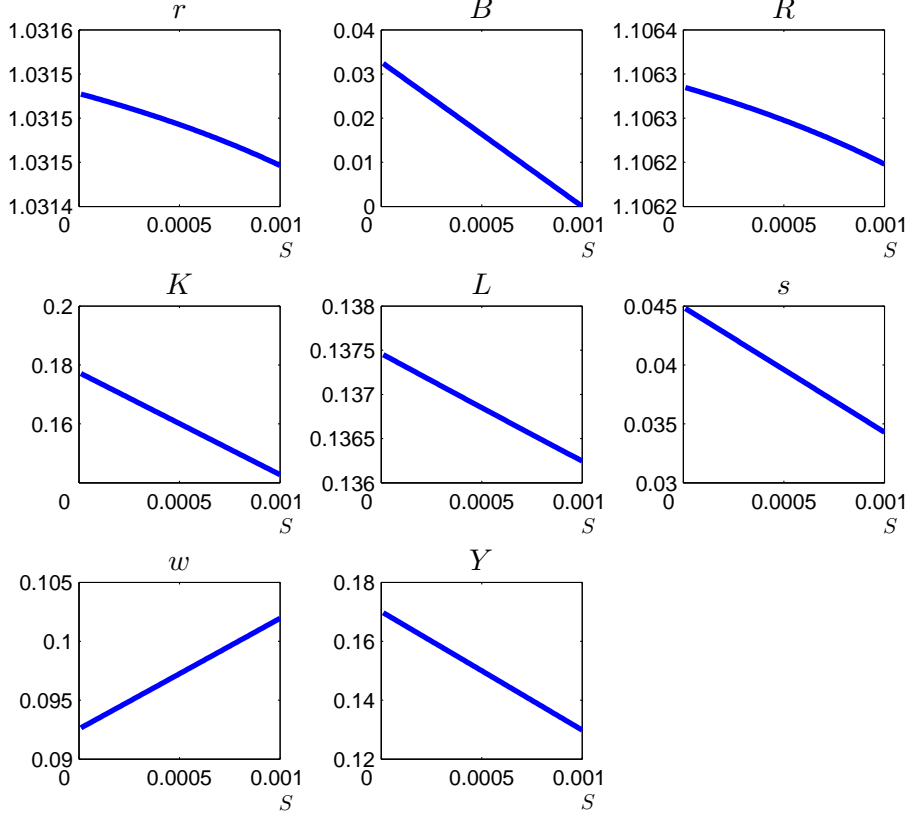
Figure 2: The steady-state equilibrium:  $S = \frac{1}{2}T$  ( $> 0$ )



shortage by issuing the government bond as its coercive power to impose tax can work as a source of trust that complements the limited commitment of the private agents. Larger tax  $T$  and bonds  $B$  imply more abundant liquidity. This leads to lower interest rates  $r$  that enable the high-productivity firms to borrow more and produce more, leading to a larger output  $Y$ .

The redistribution of resources  $S$  from entrepreneurs to workers reduces output and interest rates. Figure 2 shows the steady-state equilibrium in the case where  $S = \frac{1}{2}T$  ( $> 0$ ). The parameter values for the experiment shown in Figure 2 are the same as those in Figure 1. In this case, the total output and interest rate can both decrease as  $B$  and  $T$  increase. This result can be explained as follows. As the supply of liquidity increases with an increase in  $B$ , the interest rate  $r$  decreases. Meanwhile, the increase in  $S$  decreases

Figure 3: The steady-state equilibrium with  $T = 0.001$  and various  $S$



labor supply, owing to the income effect of  $S$  on the workers, and raises the wage rate  $w$ . The total effect of the increase in  $B$  and  $S$  is to reduce net worth of the high firms and decrease  $\hat{K}$  and  $\hat{L}$ . As production by the high firms decreases, the total output  $Y$  decreases.<sup>4</sup> We should note that the result is sensitive to the value of  $n$ . For a large value of  $n$ , say,  $n = 0.1$ , the output increases as  $T$  and  $S$  increase, because the liquidity effect is dominant.

These experiments show that the income effect of the lump-sum subsidy on labor

<sup>4</sup>In the case where the workers' utility is a Greenwood–Hercowitz–Huffman (GHH) type function, or the labor supply does not depend on workers' incomes, the accumulation of public debt increases the output, even if  $S > 0$ . This result, however, is not reported in this paper. In the case of GHH utility, the lump-sum subsidy  $S$  has no income effect on the labor supply. Thus, the liquidity effect of the government bond becomes dominant such that total output increases as  $B$  and  $T$  increase.

supply is one of the crucial factors that generate the negative effect of debt and subsidy on output and interest rate. In short, the public debt per se can enhance efficiency in our economy, whereas the redistribution from entrepreneurs to workers may depress output through the income effect on labor supply. Figure 3 reinforce this observation. I fix the amount of the lump-sum tax and observe the steady-state equilibria corresponding to various values of the lump-sum subsidy. Figure 3 shows that the output and interest rate decrease as  $S$  increases. The result is robust against perturbations of the parameter values for a small  $T$ .<sup>5</sup> This result implies that as the wage rate  $w$  increases with an increase in  $S$  through the income effect on labor supply, the net worth of the high firms is depressed and output decreases. The interest rate also decreases, because the decrease in the high firms' net worth reduces the demand for borrowing.

These results imply that persistent stagnation may not be caused by a large debt per se, but possibly by a large redistribution from the entrepreneurial sector to workers, such as through social security spending. Thus, a reduction of the outstanding amount of public debt may not be necessary to restore economic growth. Instead, it may be necessary to decrease the extent of redistribution through social security and/or impose a larger tax burden on the beneficiaries of social security spending.

## 4 Conclusion

I demonstrate in this paper that expansionary fiscal policy associated with higher public debt may decrease output and lower interest rates for some parameter values. Public debt per se may not decrease the output, but the redistribution from entrepreneurs to workers could do so. This is because a large public debt may enhance efficiency by providing liquidity, whereas the transfer of resources by the government from productive agents to workers may reduce labor supply by the income effect, leading to higher wage rates, and thus, reduced net worth of productive agents. The exercise in this paper implies that fiscal consolidation, that is, reduction of redistribution and public debt, may enhance economic growth and raise the real interest rate.

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<sup>5</sup>It is shown that for a large  $T$ , the output increases as  $S$  increases.



## Appendix: Solution for the steady state

We first describe the method to arrive at the solution for the system of equations (20)–(31).  $T$  and  $S$  are given. First, we take  $\tilde{S} = S/w$  as given. Then the variables are described as functions of  $(K, \hat{K})$ .  $K'$  is given by (27);  $w$  is determined by (31), that is,

$$\frac{1 - \omega\tilde{S}}{1 + \omega} = z^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} \hat{K} + \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} K',$$

implying that  $w$  is given by

$$w(K, \hat{K}) = (1 - \alpha) \left( z^{\frac{1}{\alpha}} \hat{K} + K' \right)^{\alpha} \left( \frac{1 + \omega}{1 - \omega\tilde{S}} \right)^{\alpha};$$

$r = r(K, \hat{K})$  is given by (21);  $B = B(K, \hat{K})$  is given by (20);  $R = R(K, \hat{K})$  is given by (22);  $W = W(K, \hat{K})$  is given by solving (25) as follows:

$$W = \frac{1}{\beta} \left[ K - \frac{w\tilde{S}}{r - 1} + \frac{\beta R - 1}{R - 1} nT + \frac{\{(1 - n)\beta r + n\}}{r - 1} T \right];$$

and  $s$  is given by solving (26) as follows:

$$s = \frac{\hat{K} + \frac{\beta R - 1}{R - 1} n\hat{k}T}{\hat{k}\beta W}.$$

Then, the variables  $(K, \hat{K})$  are determined as the solution to (23) and (24), given  $\tilde{S}$ . Finally,  $\tilde{S}$  is determined by

$$w\tilde{S} = S.$$

### Steady state with $T = S = B = 0$

The solution is different for the steady state with  $T = S = B = 0$  that is determined by the following system of equations:

$$r = \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1-\alpha}{\alpha}}, \quad (32)$$

$$R = \frac{\left(1 - \frac{\theta}{\alpha}\right) z^{\frac{1}{\alpha}}}{1 - \frac{\theta}{\alpha} z^{\frac{1}{\alpha}}} r, \quad (33)$$

$$W = [Rs + r(1 - s)]\beta W, \quad (34)$$

$$sW = \gamma R\beta sW + (1 - \gamma) \frac{n}{1 - n} r\beta(1 - s)W, \quad (35)$$

$$K = \beta W, \quad (36)$$

$$\hat{K} = \hat{k}\beta sW, \quad (37)$$

$$K' = K - \hat{K}, \quad (38)$$

$$\hat{L} = z^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} \hat{K}, \quad (39)$$

$$L' = \left( \frac{1 - \alpha}{w} \right)^{\frac{1}{\alpha}} K', \quad (40)$$

$$L = \frac{1}{1 + \omega}, \quad (41)$$

$$L = \hat{L} + L'. \quad (42)$$

**Solution:** We denote  $x$  as

$$x = \frac{\left(1 - \frac{\theta}{\alpha}\right) z^{\frac{1}{\alpha}}}{1 - \frac{\theta}{\alpha} z^{\frac{1}{\alpha}}}.$$

We take  $r$  as given. Then  $R = xr$ .

From (34) and (35), we have

$$s = \frac{\gamma xs + (1 - \gamma) \frac{n}{1 - n} (1 - s)}{sx + 1 - s}. \quad (43)$$

$s$  is decided by (43). From (34), we have

$$r = \frac{1}{\beta(sx + 1 - s)}. \quad (44)$$

$r$  is given as the solution to (44).  $R$  is given by (33). Then  $w$  is given by (32). From (36)–(41), the last equation (42) can be written as

$$\frac{1}{1+\omega} = \left[ z^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} \hat{k}\beta s + \left( \frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} (1-\hat{k}s)\beta \right] W \quad (45)$$

that determines  $W$ .

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